

GoGeometry Problem 350

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$QS=HP$ (common tangents of circles 2,3). We know of 349 problem that

$$EG = FH = DP = DN = RM = QR$$

$QS=HP \Rightarrow QR+RS=HF+FP \Rightarrow RS=FP$ But $RS=RN$ (common tangents of circle 3) so $FP=RN$

$CT=CF$ (tangent to circle 1), $CP=CK$ (tangent to circle 3) therefore $CF-CP=CT-CK \Rightarrow$

$\Rightarrow FP=TK$. Because $FP=RN$. finally $RN=TK$. But $BN=BK$ (tangent to circle 3)

So $BN-RN=BK-TK \Rightarrow BR=BT \Rightarrow$ Triangle BRT is isosceles, so bisector BI the angle DBC is mediator of RT . So $\angle LRT = \angle LTR = \angle x$, $\angle BRL = \angle \omega$ and $\angle ELR = \angle 2x$

because the quadrilateral $EBLR$, $\angle BEL = \angle BRL = \angle \omega$ is inscribable. So

$$\angle ABD = v = \angle ELR = \angle 2x = 2 \angle ETR$$

o.e.d

FIGURE

